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Mathematics Education

Dialogues

Volume 2, Issue 3

Groping and Hoping for a Consensus on Calculator Use

The first four-function and scientific handheld calculators appeared in the early 1970s. Their appearance gave rise to simplistic yes-no articles in newspapers and magazines. Yet those opinions had little effect because cost, fragility, and short battery life limited calculator use.

By the early 1980s, those deterrents lost force because of the appearance of solar-powered, hard-case, four-function and scientific calculators costing less than \$10 and \$15, respectively. And so the new generation of calculators began to be used.

In 1985, the first user-friendly calculators appeared that could graph functions. Like their simpler counterparts, these calculators were too expensive to be widely adopted when they first appeared, but today many high schools require them for all or virtually all their students. The use of these calculators in secondary school has not generated as much controversy as the use of simpler calculators in elementary school, and they are required on many college-entrance tests.

More recently, user-friendly calculators have appeared that can solve literal algebraic equations, manipulate algebraic expressions, differentiate and integrate, and solve systems of equations. Just as their earlier counterparts raised questions about the amount of paper-and-pencil arithmetic a person needs to know, these symbol manipulators force an examination of the amount of paper-and-pencil mathematics a person needs in algebra through calculus and beyond.

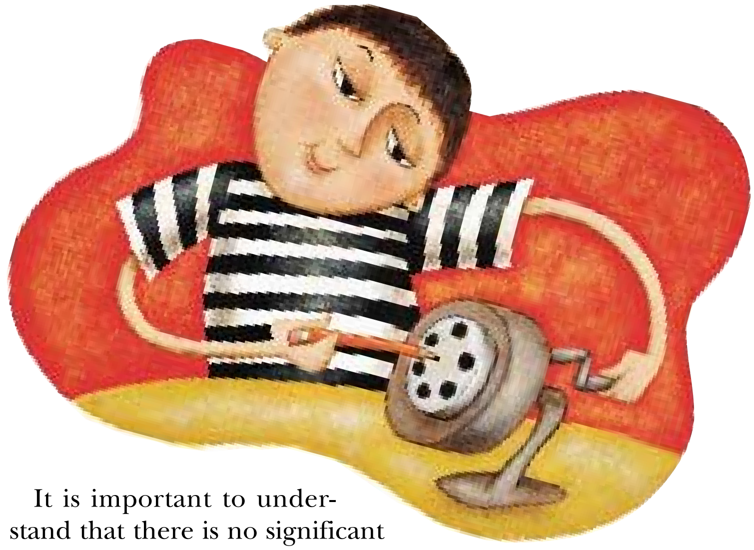
The issues relating to calculator use reach the very core of mathematics instruction. What type of understanding does one obtain through repeated application of algorithms? What new understandings, if any, can arise from calculator use, and what understandings, if any, may be lost? How important are speed and accuracy with paper and pencil when a calculator is usually faster and more accurate? What becomes obsolete because of the existence of calculators?

Though it has been more than a quarter-century since these questions were first raised, we still seem to be a long way from developing a consensus on the use of calculators in the classroom. We hope the discussion has moved away from the simplistic yes-no responses of yesteryear to a more sophisticated analysis. Resolution requires an answer to the same question asked by the mathematicians of Europe 400 years ago, when algebra as we use it today was first developed: How should we make use of this extraordinary technology to further the mathematics education of our students?

Zalman Usiskin, Editor
For the Editorial Panel

Inside this issue …

Let's Abolish Pencil-and-Paper Arithmetic



by Anthony Ralston

A paper of mine with the title above will be published later this year in the *Journal of Computers in Mathematics and Science Teaching* (see www.doc.ic.ac.uk/~ar9/abolpub.htm). In it I propose that the elementary school mathematics curriculum should not attempt to achieve any level of proficiency whatever in pencil-and-paper arithmetic.

Such a proposal may seem peculiarly perverse at a time when there is grave concern about the poor showing of American students in TIMSS (Third International Mathematics and Science Study)and when there is a feeling among many university mathematicians that their students are ever more poorly prepared in mathematics. But I mean it.

Although the problems with precollege education in the United States are multifaceted, with curriculum being only one of the possible causes of those problems, many ascribe the present situation in mathematics to a progressive "dumbing down" of the curriculum in recent years. Calculator usage, particularly in elementary school, is often seen as the chief culprit. The solution often proposed—for example, in California—is a "back to basics" approach with a ban on calculators in elementary school.

Nevertheless, my proposal is to allow full use of calculators from kindergarten on, with instruction in pencil-and-paper arithmetic replaced by a greatly increased stress on mental arithmetic for all one- and two-digit calculations. The aim would be for a curriculum at least as demanding as any currently in use or proposed, one in which students would develop their own algorithms for mental arithmetic—guided by their teachers—and in which calculators would be used not just for arithmetic but also for creative explorations and problem solving. I haven't the space here to discuss the details further, but I believe that such a curriculum, appropriately elaborated, not only would serve elementary school children better than any current one but also would prepare them better than at present for secondary school and college mathematics.

It is important to understand that there is no significant research that suggests that calculator use at any level is harmful to mathematical development or that pencil-and-paper arithmetic, a skill with rapidly declining practical value, is necessary or even particularly useful for later mathematical development. Moreover, the lack of *technique,* so often found in secondary school and university students and rightly deplored by university mathematicians, is not at all the result of calculator usage but rather the failure to develop *number sense* in elementary school and *symbol sense* in secondary school. Both number sense and symbol sense can, I believe, be better developed with a calculator-and-mental-arithmetic–based curriculum than with current curricula.

What stands in the way of developing and implementing an elementary school mathematics curriculum based on mental arithmetic and calculator usage? One barrier might be the ability of current elementary school teachers to teach such a curriculum. For this reason (among others) I advocate the use of mathematical specialists in elementary school from third grade on, if not earlier.

Then there are the political and parental barriers, which I cannot discuss here. Finally, there is the research mathematics community, which, despite some notable exceptions, understands little but inveighs a lot about the elementary school curriculum. In any event, my prediction is that until we abolish teaching pencil-and-paper arithmetic in American elementary schools, the hand-wringing at the poor performance of American students in international comparisons will be a continuing phenomenon of the American educational scene.

Tony Ralston is professor emeritus of computer science and mathematics at the State University of New York at Buffalo and an Academic Visitor in the Department of Computing, Imperial College, London. He has a long-standing interest in, and has written widely about, mathematics education.

The purpose of ***Mathematics Education Dialogues*** is to provide a forum through which NCTM members can be well informed about compelling, complex, timely issues that transcend grade levels in mathematics education. The opinions expressed in this publication are those of the writers and do not constitute an official position of the NCTM. ***Mathematics Education Dialogues*** is published as a supplement to the ***News Bulletin*** by the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1593. NCTM Reston staff: Charles Clements, Copy Editor; Sherry Grimm, Editorial Assistant; Debra G. Kushner, Graphic Designer; Andy Reeves, Staff Liaison. Pages may be reproduced for classroom use without permission.

Do We Need Calculators?

by Kim Mackey

We all use calculators in daily life; why should we forbid them in school mathematics classes, hobbling our children as we would never consider hobbling ourselves?

Even so, I believe that calculators are often detrimental to the teaching of mathematics. As the mathematician Ralph Raimi has written, “Education is not imitation of life; it is an artificial process designed to put ideas into the mind and not answers on paper.” At the grades K–6 level, and given the knowledge base of our average teacher, calculators produce only answers.

From the TIMSS results it is clear that mathematical competence at the grades K–6 level does not require calculators. Two of the highest-achieving countries at the fourth- and eighth-grade levels, Singapore and Japan, use calculators sparingly in elementary schools.

At the grades 7–16 level we might do well to heed the voices of those with the most experience with calculator usage. John Duncan, a mathematics professor at the University of Arkansas and an early advocate for technology in a college setting, wrote these cautionary words in 1995 (*American Mathematical Monthly* 102, p. 194): “Some of us who were very early to use technology to alleviate drudgery, to visualize graphs and surfaces, to conduct helpful experiments, etc., are now alarmed at its use as a substitute for thinking. It even seems to deter problem solvers from producing general mathematical proofs by holding their focus to computing a few numerical examples.”

Overseas, Great Britain has been at the forefront of using calculators in both elementary and secondary school settings since the 1982 Cockcroft Report. Anthony Gardiner, a mathematician from the University of Birmingham, has graded thousands of competition papers and has developed an excellent sense of the shift in mathematical capabilities of students in his country over the past fifteen years. According to Gardiner (personal e-mail), the worst effects of calculator usage are the following:

1. The loss of experience in *simplifying* and the consequent loss of the student’s (and the teacher’s or examiner’s) expectation that expressions should have any *meaning*.
2. The *destruction within half a generation* of a hard-won, effective algebraic symbolism, developed and proved over centuries, capable of being manipulated as a “calculus” for exact numerical and symbolic calculations, and its replacement by slavish verbatim copies of what appears in calculator displays.
3. The collapse within ten years of arithmetical fluency within the very best students, with the resulting loss of meaning for symbolic generalizations of numerical expressions.
4. The loss of all attempts to teach pupils to present solutions in forms that others can make sense of, and the decline into mere personal jottings en route to an answer, which is related—I suspect—to the next effect.
5. The two most damning outcomes of post-Cockcroftian innovations:
   a) The astonishing switch from solving simple problems (i.e. *methods*) to caring only about *answers* (i.e., things that appear in the display of a calculator)—exactly the opposite effect of what the innovators claimed they wanted—and so horribly widespread that no one can pretend not to know what has happened;
   b) The inability of students to solve two-step problems because teachers and examiners have learned to accept mere answers, since psychologically it is almost impossible to train students who are expected to use a calculator to write anything else down on paper.

One might assume from what I have written and quoted that I am an antitech Luddite who forces his students to do thousands of long-division problems by guttering candlelight. Not so. I have a dozen computers in my room and ten TI-92s for students’ use. But I think too much of my students and the mathematics they need to learn to condemn them to a black-box paradise of mindless button pushing merely for the sake of being on the cutting edge of the mathematics reform movement.

Kim Mackey has taught mathematics and science in Alaska for twelve years. In 1994 he received a Distinguished Teacher award from the White House Commission on Presidential Scholars, and in 1998 he was honored as the National Academic Decathlon Coach of the Year.

Correction: The quote from Judy Sowder in the “Reactions to Tracking” section on page 14 of the April 1999 issue of *Dialogues* contains an error. Here is the quote as it should have appeared, with the corrected word in boldface.

An article in the November 1998 issue of the *Journal for Research in Mathematics Education* (JRME) offers some interesting perspectives on tracking students based on ability levels. The Israeli researchers, Liora Linchevski and Bilha Kutscher, investigated the effects of placing students in mathematics classes that were tracked by ability levels (homogeneous groups) and those in which students of all ability levels were in the same class (heterogeneous groups). They found that the achievements of high-ability students in heterogeneous classes were not compromised but that the achievements of average- and low-ability students were significantly lower in **homogeneous** classes.

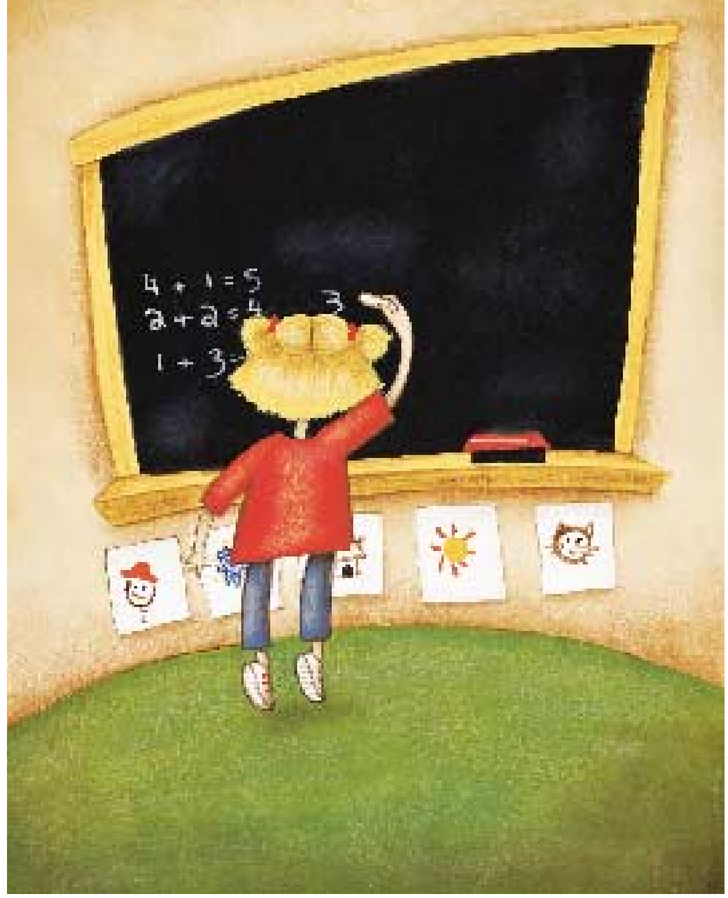
Judith T. Sowder
University Teacher, California; Editor, *JRME*

How Our Readers Feel about Calculators

by Cynthia Ballheim, *For the Editorial Panel*

In anticipation of this issue of *Dialogues,* we invited our readers to share their "preactions" concerning the way equity considerations affected how their schools (or districts) dealt with calculators. Of the more than 360 responses received, only one person thought that equity issues did not apply in his school, because the school was against calculator use. The rest of the respondents informed us that their schools supplied calculators to all students who needed them and, in fact, issued them along with textbooks. They agreed, however, that some students in the same class did have more powerful calculators than others and that this is where inequity existed.

Personal views on the use of handheld calculators were very similar. No one suggested that calculators should not be used in schools. Most recommended that the mathematics that is taught should be revised to acknowledge the power of calculators but that calculators should be used only after teachers were knowledgeable about the equipment and students had received appropriate instruction. The majority believed that calculators should be used only after students had learned how to do the relevant mathematics without them. If grade restrictions were to be put on calculator usage, our readers were split on just when this was to occur. One-third of those who responded to this question thought that calculators "should always be available" for student use without any kind of restriction. If we interpret the response "always, with no restriction" as an "anytime after kindergarten" response, we have the results from our respondents shown in figure 1.



Calculators should be available after grade:

K	69	Grade 6	14
Grade 1	2	Grade 7	21
Grade 3	2	Grade 8	38
Grade 4	16	Grade 9	4
Grade 5	8	Grade 10	6

Fig. 1

In a nutshell, our readers told us that calculators are powerful tools when used appropriately and that they should be used in all grades where problem solving is the main focus. They further stated that calculators allow students to explore and try new ideas and that they relieve students of cumbersome computation, allowing them to concentrate on more meaningful mathematical activities. Readers said that as a tool, a calculator can enhance and enrich students' competencies. Readers advised caution, however, and strongly advocated education on how to use calculators. Some of the responses follow.

There needs to be a balance between mental, paper, and calculator work.

Cyndy Montes
Grade 8 Teacher, New Jersey

They should be allowed for visual demonstrations and should serve to expedite calculations that students can already do.

Maria J. Vlahos
Grades 9–12 Teacher, Illinois

Calculators and other technological resources can be incredibly powerful tools when used in the classroom for prediction and demonstration. It is imperative, though, that teachers give careful thought about how they can be used most effectively as a tool rather than a crutch.

Lora C. Pitman
Grades 9–12 Teacher, New Jersey

Calculators are tools that students need to know how to use.

Janice James
Grades 1–3 Teacher, Kentucky

Calculators can be used to teach basic concepts to students before getting into the details of how.

Ronda Collette
Grades 9–12 Teacher, Colorado

I have little tolerance for the equity issue. If kids can prance around with sculptured nails and pierced body parts, they can afford a calculator. In addition, if they paid for their calculators themselves, they'd take better care of them.

Alice Hess, I.H.M.
Grades 9–12 Teacher, Pennsylvania

Students will attempt a multistep, difficult problem with a calculator but not without one.

Mary M. Swart
Grades 4–9 Teacher, New York

Calculators make higher math accessible to many students who have trouble with arithmetic.

Laura Reed
Grades 9–12 Teacher, Vermont

It is important that children develop math sense and understand the reasonableness of an answer.

June Lange Prewitt
Kindergarten Teacher, Missouri

Use the technology!

Don Cardinal
Grades 10–12 Teacher, Pennsylvania

A waste of my time. NCTM should concentrate on promoting qualified math teachers in all classrooms and less on this nonsense.

David Detje, FSC
Grade 7 Teacher, New York

We must put technology into the hands of our children. Then we must teach them how and when to use calculators.

Kim P. Loucks
Grade 8 Teacher, New York

Calculators should be given to students before paper and pencils.

William J. Hewitt
Grades 9–12 Teacher, New Jersey

People who lack good grammar skills can be misled by the corrections offered by computer grammar checkers. People who lack good math skills (including interpreting results) can be misled by what their calculators tell them. Calculators must be used thoughtfully.

Wayne A. Williams
Grades 6–8 Teacher, North Carolina

Numeracy and conceptual understanding are developed through a variety of modalities.

Muriel Aynanaba
Grade 5 Teacher, California

I have high school students who literally can't tell me 7×8 without a calculator.

Jill Thompson
Grades 9–12 Teacher, Missouri

This issue has been the burning one for centuries. Calculate or compute? That is the question.

Robert Foote
Grades 6–7 Teacher, Illinois

There are times when calculators need to be held back temporarily—for example, matrix inverses, graphing techniques.

Raymond Whipple
Grades 9–12 Teacher, Massachusetts

The power of calculators is changing so rapidly. Teachers must upgrade the curriculum to keep up.

Charles Mitchell
Community College Teacher, Illinois

Calculators allow for the exploration of number patterns and use of real-world data. Calculators are part of everyday life. We cheat students if they are not allowed to use them.

Lucy Hahn
Grade 2 Teacher, Idaho

It is necessary to learn a variety of algorithms as well as use a calculator.

Sylvia Linda Cotter
Grades 4–5 Teacher, Ohio

Calculators synthesize concepts and applications that are both vital components to internalize a "sense" of mathematics.

Grace Cavallo
Supervisor, New York

Lower-ability math students need to have a greater availability of calculators than others.

Steve Peterson
Grade 6 Teacher, Minnesota

Certain skills that should not require a calculator are still useful, but many other skills are antiquated and obsolete.

Steve Lieberman
Grades 8–12 Teacher, California

Our rural county still does not allow calculators to be used, because of local objections to their use on state tests.

Mata J. Banks
Grade 8 Teacher, Tennessee

"Students must be prepared for the twenty-first century" is the current mantra in support of calculators in the classroom. Using technology is easy to learn and certainly does not require the hours of practice that we give our children. Creating technology is the real challenge. This creativity springs from an understanding of mathematical ideas and algorithms, which the prevailing over-reliance on calculators stifles.

Diane Hunsaker
Tutor, California

The resourceful teacher can assess a student's understanding of concepts and even computational skill without denying calculator usage.

Craig Russell
Grades 8–12 Teacher, Illinois

To educate math students without calculators and computers is to deprive them of a rich experience in complex problem solving. However, two things must be remembered. First, basic skills such as mental calculations and pencil-and-paper calculations must be developed and maintained. Second, teachers must receive training so they become calculator literate before calculators are used in the classroom.

Holt Zaugg
Grades 7–12 Teacher, Alberta

The Research Backs Calculators

by Donald J. Dessart, Charleen M. DeRidder, and Aimee J. Ellington

Calculators are an essential technological device in our society. Schools have the duty to provide instruction for the appropriate and effective use of calculators. Most teachers agree that after students master computational skills, calculator use aids in checking computation and facilitates problem solving. We, however, maintain that calculators should be an integral part of mathematics instruction including the development of concepts and computational skills, and the research supports our position.

Hembree and Dessart (1992) reported the findings of a meta-analysis of the effects of precollege calculator use. This research analyzed results from eighty-eight studies focused on students' achievement and attitude. Each study involved one group of students using calculators and another group having no access to calculators. From their analysis, Hembree and Dessart concluded that the calculator did not hinder students' acquisition of conceptual knowledge and that it significantly improved their attitude and self-concept concerning mathematics.

Smith (1997) conducted a meta-analysis that extended the results of Hembree and Dessart. Smith analyzed twenty-four research studies conducted from 1984 through 1995, asking questions about attitude and achievement as a result of student use of calculators. As in the Hembree and Dessart study, test results of students using calculators were compared to those of students not using calculators. Smith's study showed that the calculator had a positive effect on increasing conceptual knowledge. This effect was evident through all grades and statistically significant for students in third grade, seventh through tenth grades, and twelfth grade. Smith also found that calculator usage had a positive effect on students in both problem solving and computation. Smith concluded that the calculator improved mathematical computation and did not hinder the development of pencil-and-paper skills.

A recent large study examined effects over a longer term. The purpose of the project, Calculators in Primary Mathematics, funded by the Australian Research Council, Deakin University, and the University of Melbourne, was to have primary and elementary school students explore and develop number sense using calculators before standard algorithms were taught (Groves and Stacey 1998). It involved one thousand students and eighty teachers over a three-year period. The performance of students in the project was compared with that of a control group for the same schools using a written test, a calculator test, and an interview. The results showed that the project students performed better overall on a wide range of items including place value, decimals, negative numbers, and mental computation. No detrimental effects of calculator use were observed.

Until curricular innovations such as that tried in Australia are implemented, we believe that students and teachers should distinguish among three tools of computation: mental arithmetic, pencil and paper, and calculators. For example, we would chastise any student who reaches for the calculator to find 3×4; we would suggest pencil and paper for calculating 27×340; and we would insist on using the calculator for 2.7568×345.8972 after the student estimates mentally an answer of 900 (3×300).

In conclusion, we recommend that schools strongly encourage the use of calculators in all aspects of mathematical instruction including the development of mathematical concepts and the acquisition of computational skills. We believe that calculator education is an obligation of schools to our society where calculators are in common, daily use.

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Donald Dessart

Charleen M. DeRidder

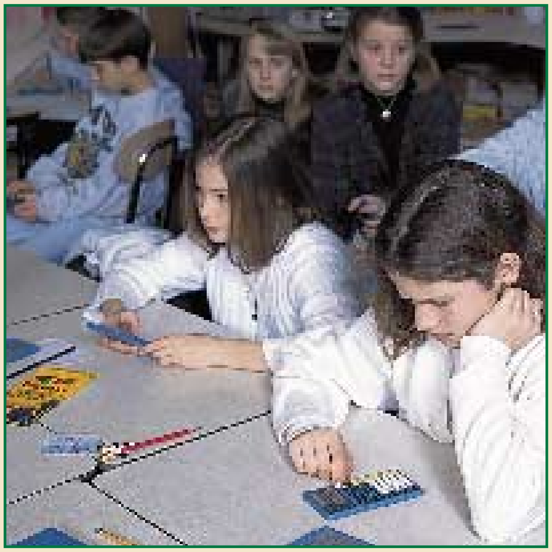
Aimee J. Ellington

Donald J. Dessart has been a professor of mathematics and mathematics education at the University of Tennessee—Knoxville (UTK) since 1962. Charleen M. DeRidder is an adjunct professor of mathematics education at UTK and supervisor of mathematics for grades K–12 in the Knox County Schools of Tennessee, and Aimee J. Ellington is an instructor of mathematics at the University of Tennessee at Chattanooga.

Ensuring That All Children Are Powerful Technology Users

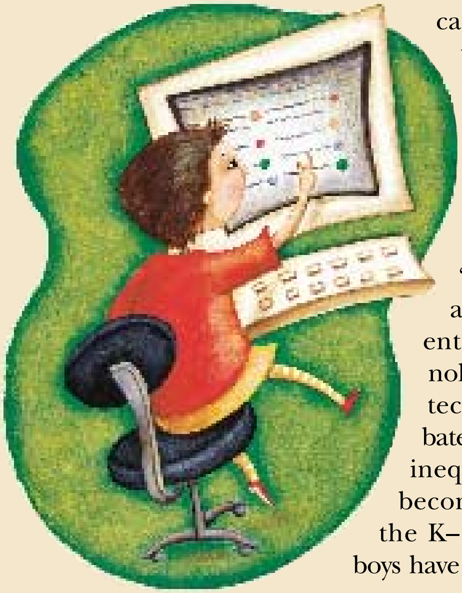
by Charlene Morrow

A calculator is not only an essential tool in today's world but a bridge for young children into the world of scientific technology. Our reluctance, as educators, to teach children to use calculators as a mathematical tool promotes inequities that will later affect students' access to college courses and to work. When we don't structure equitable opportunities for children to use technology, we allow social forces to take over, leading to more comfort with technology on the part of, and usage by, males than females and more comfort on the part of those students whose families are willing and able to buy and use technology.



Historically, the use of tools has empowered tool users to gain control over their environment and to become experts, with associated economic benefits. In fact, the use of tools, along with language, is the essence of being human. The use of technologically advanced tools, especially those associated with paid work rather than housework, has been the province of males. We will need to work hard to make sure that these cultural factors do not prevent females (and other historically disenfranchised groups) from having equal access to important modern-day tools. The American Association of University Women (AAUW), in its recent publication *Gender Gaps: Where Schools Still Fail Our Children,* has identified the use of technology as having among the most significant gender gaps of any area in school.

Lest you are about to jump up and toss calculators into the classroom, let me caution you that this is where things get tricky. What if calculator use does not promote a "girl friendly" learning environment? According to the AAUW again, "Girls have developed an appreciably different relationship to technology than boys … and technology may exacerbate rather than diminish inequities by gender as it becomes more integral to the K–12 classroom." More boys have and use technological tools and toys. Boys more often perceive themselves as going into careers, such as engineering, that require technology like calculators and computers. They can envision a payoff for learning to use these tools. We need to be sensitive to past and present inequities when structuring opportunities for all children.



In our own program, SummerMath, a four-week program for a diverse group of high school girls of varied mathematical experiences, girls enthusiastically take to technology—both calculators and computers. But we are not satisfied with girls using technology only when others say to do so. Our objective is for girls to be powerful users of technology—initiators of technology use. This summer we are recognizing the importance of the ownership of tools by buying each student a graphing calculator to take home.

Unstructured experiences with technology will simply preserve the social order—opportunities need to be structured in order to be equitable. The AAUW has many suggestions on this topic. It is incumbent on us as educators to ensure that these structured opportunities occur in school. Although mathematics educators argue about whether or not to teach and encourage children to use calculators from an early age, some children—more males than females—will be learning to use them anyway and thus will be ready for a wider array of college andcareer opportunities.

Charlene Morrow, cmorrow@mtholyoke.edu, is the co-director of the SummerMath program at Mount Holyoke College, South Hadley, MA 01075-1441, where she is a faculty member in the Psychology and Education Department. She is a past chair of the NCTM Committee for a Comprehensive Mathematics Education of Every Child and a coauthor of the new book Notable Women in Mathematics. *Her current work involves the exploration of mathematical ideas in quilts and in origami.*

A Calculator Tour around Canada and the United States

by Johnny Lott, *For the Editorial Panel*

Although there may be controversy in some segments of the population about student use of calculators in learning and doing mathematics, that controversy is not apparent in most policy documents at the provincial or state level. Most policies are very similar, only with some having more specificity than others. The adapted samples below are representative of the genre.

Canadian Viewpoints

The intelligent use of a graphing calculator should be emphasized at all times. Students should think about the best method of solution before they reach for their calculators. Up to 15 percent of the provincial exam will contain questions that are dependent on a graphing calculator. Using a graphing calculator, a student should be able to—

1. produce a graph within a specified viewing window;
2. determine an appropriate viewing window to examine a graph and change the window's dimensions;
3. use the zoom features of the calculator;
4. find zeroes and intersection points (usually to two decimal places);
5. find maximum or minimum points....

Most of the currently available graphing calculators have the built-in capabilities to do all the skills above; however, some of the older models may require programs. Calculator memories will not be cleared at the time of examination. Many graphing calculators do much more than these five specified calculator skills. Teachers are encouraged to use other graphing calculator features, such as parametric or polar graphs, to enhance the learning of mathematics in the classroom. However, teachers and students should be assured that only the specific calculator skills listed above will be required for the provincial examination.

British Columbia Graphing Calculator Resource Package for Principles of Mathematics 12, p. 5

Emphasis is placed on the role of technology and the appropriate concepts and skills related to its use. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology not only has made calculations and graphing easier but has also changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

Atlantic Canada Mathematics Curriculum Teachers Draft Guidelines, p. ii

Scientific calculators, graphing calculators and related probes, dynamic graphing software, and accessing the Internet are becoming commonplace in our students' lives. Technology makes students more powerful learners by allowing them to visually explore mathematical concepts more easily and quickly. This allows fundamental ideas to be studied in greater depth, giving students more time for exploration in the areas of data collection, data analysis, simulations, and complex problem solving. Whereas investigators once relied on their creativity and the sophistication of known mathematical models to guide them in the solution of problems, technology now provides capabilities that alter both the form and the means of solution. Calculators save time in performing complex arithmetic calculations. Graphing utilities enable students to visualize relationships and test hypotheses.

Ontario—Secondary Policy Document for Mathematics, Grades 9 and 10 (in draft form), p. 4

Overall expectations in the Ontario Curriculum, Grades 1–8, include the following:

- Use a calculator to solve number questions that are beyond the proficiency expectations for operations using paper and pencil.
- Justify the choice of method for calculations: estimation, mental computation, concrete materials, pencil and paper, algorithms (rules for calculations), or calculators.

Ontario—The Ontario Curriculum, Grades 1–8: Mathematics, Grade 8, p. 26

U.S. Viewpoints

In Illinois, Mississippi, New Jersey, and Colorado, there are specific statements about calculator use in the state standards. These standards

are more generic than some of the ones seen in the Canadian provinces above, but they mirror comparable state standards in the rest of the United States.

General Mathematics Standards

- Identify and describe patterns and relationships in actual data, as well as solve problems and predict results using algebraic methods and symbols, tables, graphs, calculators, and computers.
- Analyze, categorize, and draw conclusions about objects and spatial relationships using geometric methods and drawings, sketches, graphs, models, symbols, calculators, and computers.

Adapted from Illinois Academic Standards in Learning Areas (Draft, 1996)

Grade 8 Algebra

- Solve equations and inequalities containing rational coefficients; include real-life problem-solving situations; use manipulative materials and calculators or computers where appropriate.

Adapted from Mississippi Mathematics Curriculum Structure (1995)

Grade 8 Mathematics

- Explore linear equations through the use of calculators, computers, and other technology.

Taken from New Jersey Core Curriculum Content Standards (1996)

General Mathematics Standards

- Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper and pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

Benchmark for Mathematics Standards: Grades 5–8

- Solving simple linear equations in problem-solving situations using a variety of methods (informal, formal, graphical) and a variety of tools (physical materials, calculators, computers).

Adapted from Colorado Model Content Standards for Mathematics (1995)

(Other state standards like those above can be found at www.ccsso.org.)

In stark contrast to other state and provincial calculator policy statements is the Mathematics Framework for California Public Schools K–12. *Selected pertinent quotes follow:*

The Mathematics Standards were written with the belief that there is a body of mathematical knowledge—independent of technology—that every student in K–12 ought to know, and know well. More importantly, the STAR assessment program—carefully formulated to be in line with the Standards—does not allow the use of calculators all through K–11.

More to the point, it is imperative that students in early grades be given every opportunity to develop a facility with basic arithmetic skills without reliance on calculators.

For these reasons, this Framework recommends that calculators not be used in the classroom before grade 6.

Beyond the anecdotal, there is also the input from the Third International Mathematics and Science Study (TIMSS). For the 8th grade assessment, the majority (>50%) of the students from three of the five nations with top scores (Belgium, Korea, and Japan) never or rarely (once or twice a month) used calculators in mathematics classes. In contrast, the majority of students (>65%) from 10 of 11 nations, including the United States, with scores below the international mean, used calculators almost every day or several times a week in mathematics classes (Beaton, Mullis, Martin, Gonzalez, Kelly, and Smith 1996) While such data do not prove that calculator usage is damaging to the development of mathematical skills, it would be folly to ignore this.

Taken from Mathematics Framework for California Public Schools K–12 (draft, 1998)

All the standards above relate to precollege mathematics. The American Mathematical Association of Two-Year Colleges has taken a strong stand on the use of calculators and technology, as seen in the following:

Basic Principle

- The use of technology is an essential part of an up-to-date curriculum.
- Faculty and students will make effective use of appropriate technology. The technology available to students should include, but not be limited to, that used by practitioners in the field. Faculty should take advantage of software and graphing calculators that are designed specifically as teaching and learning tools. The technology must have graphics, computer algebra, spreadsheet, interactive geometry, and statistical capabilities. (p. 5)

Standard I–6: Using Technology

- Students will use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results. (p. 11)

Standard P–1: Teaching with Technology

- Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction. (p. 15)

Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus (1995)

Questioning the Use of Calculators in the Elementary Grades

by Frank Wang

I was probably one of the first elementary school students ever to use a calculator. As a second grader in elementary school, I remember marveling at my father's Hewlett Packard HP-35 electronic calculator. I believe this calculator was the first commercially available pocket scientific calculator. The HP-35 was introduced with much fanfare in 1972 and was so named because it had 35 keys. My father, an associate professor of civil engineering at the Rensselaer Polytechnic Institute (RPI) in Troy, New York, bought one of the calculators for $350, no small sum considering my father's annual salary was only $15,000.

Prior to purchasing the HP-35, my father owned a large, cumbersome mechanical calculator. Even though he himself invested a significant amount in these labor-saving devices, my father still insisted that I learn basic computational skills through labor-intensive methods: through memorization and, yes, drill. He didn't, however, forbid my use of the calculator.

Frank Wang is president of Saxon Publishers. He received his Ph.D. in pure mathematics from MIT in 1991.

I was allowed to use the calculator only to check answers to my homework. Being able to hold and use this precious and unique device, albeit in a very controlled setting, was powerful motivation for me to do my mathematics homework.

My present opinions on calculators stem from these early experiences. Therefore, at the elementary school level, I believe that the use of calculators should be restricted and controlled. It is not until the high school grades that I believe students should be allowed to use calculators in an uncontrolled setting (meaning that students have calculators readily available and exercise their own discretion on when and how to use them).

Do I think calculators have value in the classroom? Yes. I think calculators can serve two main purposes: (1) reducing the time spent performing tedious calculations and (2) illustrating concepts. For example, I don't object to students using a calculator to calculate to several decimal places the area of a circle. A typical scientific calculator uses up to 10 decimal places in a calculation. I also think calculators are useful in illustrating concepts—for example, the concept of exponential growth: repeated multiplication by 2 can rapidly result in a large number. The calculator can show this growth easily and efficiently.

Note that both examples cited are situations that arise in the later elementary school years. My concern about calculators belies a much greater concern of mine: the teaching and practice of very basic mathematical concepts. I repeat here one celebrated example cited by former Assistant Secretary of Education Diane Ravitch in her book *National Standards in American Education*. Ravitch noted that on the 1986 NAEP (National Assessment of Educational Progress), no 9-year-olds, fewer than 0.5 percent of the 13-year-olds, and only 6.4 percent of the 17-year-olds could solve problems like this one:

> Christine borrowed $850 for one year from the Friendly Finance Company. If she paid 12% simple interest on the loan, what was the total amount she repaid?

More recent national and international tests reveal that students today are still unable to solve such problems. Would having a calculator have helped students solve the problem? I contend not.

The problem is not whether elementary school students should be allowed to use calculators. The problem is that our elementary school students cannot solve problems involving basic mathematical concepts such as percents, ratios, and proportions. If someone can show me that using calculators can help elementary school students achieve a mastery of these basic mathematical concepts, I would heartily welcome them; otherwise, I suggest the same sort of caution and care with calculators that my father exercised with me.

Calculators at the Elementary School Level? Yes, It Just Makes Sense!

by Randall I. Charles

Calculator use in the elementary grades does make sense. In fact, it is essential to attaining key goals for the mathematics education of young children.

We use calculators in the elementary grades because not using them is almost certain to lead to the development of habits that are counterproductive to the development of number sense, problem solving, and positive dispositions.

Number sense is a foundation for early success with mathematics. Calculators can be used as a tool to help in developing the conceptual understandings and abilities that underlie strong number sense. Calculators are particularly powerful in enabling children to make and test conjectures and generalizations related to numbers and operations. For example, making and testing conjectures about counting patterns helps children understand number relationships, develops flexibility with numbers, and promotes the development of mental and paper-and-pencil computational strategies.

Problem solving is a principal reason for studying mathematics. The use of calculators allows realistic data to be used as problem contexts, problems whose solutions are within the conceptual grasp of children but whose computational demands are not. The use of realistic data is motivational and helps children see connections between school mathematics and the mathematics used in the world.

Positive dispositions are fundamental for success in mathematics. Children's lifelong beliefs about calculator use are shaped to a great extent by their mathematical experiences in the early grades. At the same time we are helping children develop accurate and efficient mental strategies and paper-and-pencil calculation strategies, it is also our responsibility to help children become responsible users of calculators. Children must learn that there are times when it makes sense to use a calculator and times when it does not.

Randy Charles is a professor in the Department of Mathematics and Computer Science, San Jose State University, San Jose, California. He has been involved in curriculum development at the elementary grades for nearly twenty years. Developing a vision and a plan for calculator use at the early grades has been part of his professional effort during these years.

Thanks and an Announcement

Dialogues began in the summer of 1997 as an idea of some members of the NCTM Board of Directors who were concerned that there was no NCTM periodical in which important controversial issues that cover all the grades could be discussed in depth. The Board immediately approved the idea of two trial issues in the 1997–98 school year and—in an unprecedented move—placed three Board members on the six-member task force to bring the idea to fruition. The strong positive reactions to those issues led the Board to convert the task force into an editorial panel and to approve three issues for 1998–99 and succeeding school years.

I have had the honor of chairing the task force and being editor for this year's issues. It has been a pleasure working with fellow Board members Peggy House and Johnny Lott and also with Cynthia Ballheim, Hung-Hsi Wu, and Barbara Marshall on the task force and then working with Peggy, Johnny, Cynthia, and Hyman Bass on the Editorial Panel. It also has been wonderful to have the assistance of the NCTM staff with design and other publication issues. And we have all very much appreciated the responses that you, the *Dialogues* readers, have made. Without your contributions, there can be no dialogue and so there can be no *Dialogues.*

We are pleased that Johnny Lott was asked—and has agreed—to be the editor of *Dialogues* for the next two years. He will be assisted next year by an editorial panel consisting of Cynthia Ballheim, Paul Zorn, and Frank Lester.

Zalman Usiskin, *Editor*

Tools for Mathematical Understanding in Middle School

by Perry Montoya and Vicki Graber

Long before the high-tech calculators of today, Mesa School District conducted research that showed how calculators were necessary tools, like rulers and compasses, and how they could assist students in the process of solving complex mathematical problems. A threefold plan was developed that would provide training for specialists and teachers, assist in the implementation of calculators in the classroom, and support ongoing use of calculators in the classroom to promote students' mathematical thinking.

First, the district-based specialists in mathematics were trained on the use of various calculators (four-function, fraction, and graphing calculators). These specialists, along with the director of mathematics, trained teachers at schools on a specific calculator along with appropriate activities where calculators could be used to promote and enhance students' mathematical thinking. Teachers were taught that if they were teaching a mathematical concept where they did not want the computation to interfere with learning the concept, then it was appropriate to use the calculator to facilitate the instruction and learning. As the teachers were trained, they received a class set of calculators along with an overhead projection calculator.

Second, teachers receive support as they implement the use of calculators. This is done through many models of professional development, such as miniconferences on Saturdays, workshops after school, specialists providing demonstration lessons in the classrooms, and the mathematics leadership at each site receiving additional training in the use of calculators throughout all standards-based lessons. The mathematics leadership can then furnish on-site support to all mathematics teachers. We have found that the key to appropriate calculator use in the classrooms is excellent professional development for teachers and access to calculators at any time throughout the day.

Third, support for the ongoing use of calculators is always evident. Currently, we are training our teachers in grades 7–9 on the TI-73 calculator, and at the end of the training they will receive a classroom set of calculators along with the overhead projection calculator. Future purchases will include Ranger probes so that "real time" data may be observed, recorded, and manipulated through investigations. In our science classrooms, teachers are using the TI-82 calculators with probes to experiment with data involving such investigations as heart rate, pressure-volume relationship in gases, acid-base titration, light intensity, and motion. When students work with data analysis and the resulting statistics, the calculator is used as a tool for reasoning and for analyzing and interpreting data. Students can see the connections between the mathematics and the science classroom. There are always ample opportunities for all teachers to receive introductory or advanced training on using the calculator as a tool to promote mathematical thinking.

Calculators give students an opportunity to manipulate large numbers while solving higher-level thinking investigations. We have found that calculators help students to think flexibly. They aid in the exploration of various techniques for solving and evaluating different situations. Also, calculators assist students with organizing and storing data as well as graphing conclusions. Students learn not only how to use the calculators correctly but when calculator use is appropriate. Students also learn when an exact answer is appropriate and when an estimate is appropriate. To simulate real life, calculators are readily available in the classroom, allowing students to focus on the mathematics rather than on the novelty of the tool. In fact, students learn when use of mental mathematics, paper and pencil, or a calculator is appropriate.

As stated in the NCTM *Professional Standards for Teaching Mathematics: Executive Summary,* "Today's students will be citizens of the twenty-first century, a century that promises to be dramatically different from the one we have known. The effects of technological innovation will continue to permeate every aspect of life.… [C]omputational skills alone do not suffice.… By the turn of the century, the need to understand mathematics in order to succeed in *all* walks of life will be without precedent" (p. 3). We in Mesa use calculators as a tool to help develop students' deeper understanding of the mathematics that will be needed in the future.

Vicki Graber is a mathematics specialist and Perry Montoya is an instructional-technology specialist in the Mesa School District, Phoenix, Arizona. The district consists of sixty-one elementary schools, thirteen junior high schools, and five high schools serving more than 70 000 students in a city of 350 000 people. They work collaboratively with many other district-based specialists to provide comprehensive professional development in using calculators as tools for mathematical understanding.

A Revolution in My High School Classroom

by Gail Burrill

From the very first day in the late 1980s when I gave my students graphing calculators, my classroom changed. The change was gradual—I started by giving tests where students turned in a "without your calculator" page before receiving the rest of the test, where a calculator was permitted. But I found myself questioning whether it was really a signal of students' ability to do mathematics if they could find the sine of $\pi/12$. What does it tell me about students' understanding that I cannot get by asking them how the sum and difference formulas can be used to derive the formula for sin 2ϕ? The mathematics had to come from the questions I asked, not the tools students use to answer.

What is different? My focus in algebra had been on routine procedures for solving equations or systems of equations, such as the linear combination method. Calculators allowed me to switch my focus to thinking and reasoning about, and with, the mathematical concepts we were studying and as a matter of fact even about some of the procedures. Access to calculators allowed us to do interesting problems we could not do before with real and messy numbers, and it allowed students who had trouble with computation or procedures to have access to another way to do those things so they could move on to the mathematics we were studying. Calculators changed the nature of my courses and the way I taught.

Calculators allow me to do old things better. Moving freely among tables, graphs, and symbols helps students understand relationships previously taught as separate entities. Students once spent a whole class filling in a table for $y = \sin x$, $y = \sin 2x$, $y = \sin 0.5x$, $y = 2 \sin x$, where finding sin $(2\pi)/3$ became the focus; the effect of the variable on the range or on the shape of the curve received a quick "look" for those who waded through the computation. With a calculator, students quickly generate enough specific examples to see and discuss the effect. The lesson is on the mathematical idea I want to teach, not on how to set it up. "List" functions help students develop the notion of variable and work with formulas in a different way.

Calculators made it possible for my students to learn new concepts that were not possible to teach before. We now study regression, correlation, and modeling in early high school mathematics. Logarithms are used to handle scaling effects and transform data, not for computation. Force fields are now part of calculus. Students use matrices to manage information and mathematical situations and to solve systems of equations. They discuss the limitations and conditions on the process, freed from routine calculations to generalize about solving systems.

Calculators also make some of the things we used to do unnecessary. An example is standard deviation. Statistics texts presented the formula for standard deviation, a statistic that describes the variation of a data point from the mean, by first using the definition that highlights the difference of each data point from the mean,

$$\sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}}.$$

In practice, to avoid tedious computation, students were instructed to use an equivalent formula,

$$\sqrt{\frac{\sum x_i^{\,2} - \bar{x}^2}{n}},$$

where the difference is not obvious. As a result, many students never really came to understand the concept.

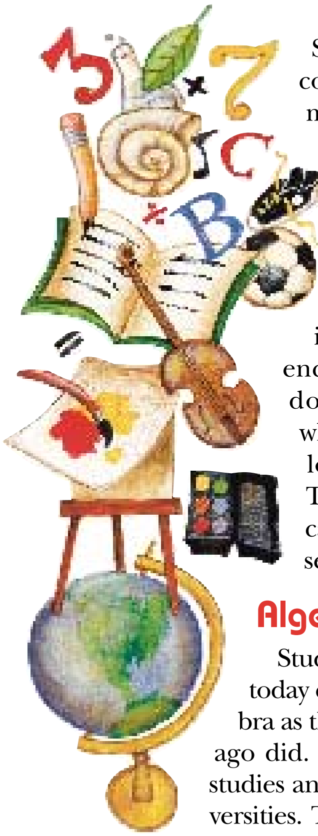
Although the content has changed, a more important change has occurred in my students. They are ready to investigate any option, they accept a challenge, and they also make mistakes. They do not always use their calculators wisely, but then most of them are doing things they never would have done before. Many of them would not even have been in that mathematics class. My students think about mathematics differently from the way I do. They reason from graphs about differences in functional values; they reason from tables about relationships; they use the replay key to sort their thinking; they reason numerically; and they confront proof in a different way from what is presented in their geometry book. They use a calculator when I least expect it on problems where I cannot see how one will help.

As in any revolution, upheaval, concern, and dissension must be expected; so, too, in the mathematics education reform movement upheaval, concern, and dissension about the use of calculators must be expected. But as a teacher, I have a responsibility to prepare my students for a world where technology is dominant and to make use of tools that will enable them to learn more mathematics and to learn it in a deeper way. And it is essential as I move with my students into the next generation of calculators that I remain focused on the mathematics I want them to learn, not on buttons to push. Giving students a state-of-the-art calculator gives them access to the power of mathematics.

Gail Burrill, the immediate past president of NCTM, is a classroom teacher from suburban Milwaukee, Wisconsin. She is currently on staff at the Center for Education Research at the University of Wisconsin—Madison and is on loan to the Center for Science, Mathematics, and Engineering Education at the National Research Council, where she is a senior program officer.

An Attempt in Sweden at Consensus

Excerpted by Zalman Usiskin from a paper given by Lars-Eric Björk and Hans Brolin presented at the University of Chicago Science and Mathematics Program's Fourth International Conference on Mathematics Education in August 1998.



This report is from the ADM (the Swedish acronym for "analysis of the consequences of the computer for mathematics education") project at the Department of Teacher Training at the University of Uppsala, Sweden. We present examples of what routine skills in the topics of algebra, trigonometry, derivatives, and functions and integrals students who intend to major in a mathematical science at university level should be able to do by hand. We also give examples of when it is appropriate to use such technological devices as graphing calculators. The task of listing important topics was carried out jointly with a large number of secondary school and university teachers.

Algebra and trigonometry

Students graduating from secondary schools today do not have the same level of skill in algebra as their counterparts of twenty or thirty years ago did. This can be seen both in international studies and in the diagnostic tests given at the universities. This is partly because of broader student recruitment and partly because algebraic manipulation has been postponed ever later in the Swedish school system. There are mainly two reasons for this. First, algebraic manipulation without understanding and application is of little value, and second, the first course, Mathematics A, which is common for all streams, contains very little content of a distinctly secondary school nature. In addition, high skill in algebra requires repeated review, which is not easy to arrange because of time shortage and modular courses. Deficient algebraic skills often constitute a severe impediment in the struggle for higher achievement levels in the understanding of concepts and in formulating and solving problems.

The following list of problems provides examples of algebraic skills that should be expected of students who have studied mathematics courses A, B, C, and D and who intend to major in a mathematical science at university level. Students with merely a "Pass" in the D course do not satisfactorily fulfill this level of algebraic skill. The problems should not be regarded as defining primary goals but rather as examples of the manipulative skills needed in order to study and understand mathematical text and to be able to solve problems.

Powers and logarithms

1. Simplify a) $\sqrt{x}\cdot\sqrt[3]{x}\cdot x^{\frac{1}{6}}$ b) $\dfrac{\left(x^2\right)^3\cdot x}{x^7}$

2. Simplify a) $\lg 70-\lg 7$ b) $\ln(1/9)+\ln\sqrt{27}$

Polynomials

3. Simplify a) $\left(x^2+1\right)(x-1)-3x^2(x+2)$
   b) $(3x+1)(3x-1)-4(2x+3)^2$

4. Factor a) $4x-x^3$
   b) $x^2(x-1)-4(x-1)$
   c) $3x^2-18xy+27y^2$

5. Complete the square x^2+6x

6. Simplify $f(x+3)-f(x)$ if $f(x)=x^2+2x-3$

Mixed rational expressions

7. Simplify a) $\dfrac{x^2-1}{x}\Big/\dfrac{x+1}{x^3}$ b) $\dfrac{3}{x}-\dfrac{2}{x-1}$

8. Simplify $\dfrac{1}{1-2a}-\dfrac{2}{1+2a}+\dfrac{6a+2}{4a^2-1}$

9. Simplify $f(\mathrm{a}+\mathrm{h})-f(a)$ if $f(x)=x^{-2}$

Solving equations and inequalities

10. Solve the equation a) $2x+3x-4=x$ b) $\dfrac{1}{2}+\dfrac{x}{6}=\dfrac{x}{3}$
    c) $2x^2+8x-42=0$

11. Solve the equation a) $\dfrac{12}{x}=0.6$ b) $\dfrac{1}{x}-\dfrac{1}{x-1}=2$

12. Solve the simultaneous equations
    a) $\begin{cases} y+x=8\\ 2x-y=7\end{cases}$ b) $\begin{cases} y=x^2-3x\\ y=2x-6\end{cases}$

13. Solve the equations a) $(x-1)\left(x^2-9\right)=0$
    b) $x^3-2x^2-3x=0$

14. Solve the equations a) $\lg x+1=\lg 7$
    b) $\ln(x+1)+\ln(2-x)=\ln(x-1)^2$

15. Solve the equations a) $x^5=2$ b) $2^x=5$ c) $10^{2x+1}=4$

16. Solve the equations $\sqrt{2x+1}=x-1$

17. Solve the inequality $2x+1\geq 4x-2$

Formula manipulation

18. Solve for x if $ax+b=cx$

19. Solve for C if $A=B+1/C$

20. Solve for p if $\dfrac{a}{1+p}=c$

21. Solve for z if $A=\dfrac{B}{(c+z)^2}$

Trigonometry

22. Trigonometry in right-angled triangles. Be able to use exact function values for 30°, 45°, and 60° and to change angles readily from degrees to radians and vice versa.

23. Triangle theorems (Area of triangle = $ab \sin C$, law of sines, law of cosines)
24. Solve basic trigonometric equations completely. For example, find all solutions for $\cos(3x + \pi/2) = 0.5$
25. Use the unit circle to demonstrate simple relationships. For example, $\sin(x + 180°) = -\sin x$
26. Be able to use

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y,$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\sin 2x = 2 \sin x \cos x, \ \cos 2x = \cos^2 x - \sin^2 x$$
$$\sin^2 x + \cos^2 x = 1$$

to rewrite expressions such as

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$(\sin x + \cos x)^2 = 1 + \sin 2x$$
$$a \sin x + b \cos x = A \sin(x + v)$$

Derivatives, plotting curves, integrals

In the parts of previous secondary school courses dealing with derivatives, curve construction, and integrals, the emphasis was largely on algebraic skills and rote procedures. Today, using the graphing calculator, algebraic formulation of problems can be complemented with numerical and graphical representation, which substantially facilitates students' understanding and permits the use of realistic mathematical models for real-life situations.

The following level of skill should be required of students who have studied courses A–D at secondary school.

Derivatives

Without calculating devices

In various applications, be able to

- realize that the value of a certain derivative should be calculated
- interpret the meaning of a certain value of a derivative.

Be able to use basic differentiation rules on elementary functions (except for inverse trigonometric functions)

$y = x^n$, $y = e^x$, $y = a^x$, $y = \ln x$
$y = \sin x$, $y = \cos x$
$y = 1/x$, $y = \sqrt{x}$, $y = 1/\sqrt{x}$, $y = x^p$

as well as linear combinations of these.

Be able to use the chain rule in such examples as

$y = \sin 2x$, $y = e^{3x}$, $y = \ln(4x)$, $y = \sqrt{1-x}$, $y = 3 \sin(2x + 4)$
$y = (\sin x)^{10}$, $y = e^{x^2}$

Be able to use the product and quotient rules in simple functions like

$$y = e^{2x} \sin 3x, \quad y = \frac{2x+1}{5-3x}, \quad y = \frac{\sin x}{\cos x}$$

With a graphing calculator

Be able to calculate the numerical value of a derivative.

Curve Construction

Without calculating devices

Be able to interpret graphs and understand the graphical significance of $f(a)$, $f'(a)$, $f(x) = 0$, $f(x) = k$, as well as the zeroes, extreme values, and concavity of f.

Understand the connection between the derivative and the appearance of the graph (the essence of traditional sign study); be able to state the equation for the tangent at a point.

Recognize and be able to sketch, with or without a table of values, simple curves for

- linear functions
- quadratic functions
- third-degree polynomials, for example, $y = ax^3 + bx$
- trigonometric functions, for example, $y = \sin x$, $y = \sin 3x$, $y = 2\sin x$, $y = \sin x + 2$, $y = \sin(x + \pi/3)$
- exponential and logarithmic functions such as $y = 2^x$, $y = e^{2x}$, $y = \ln x$
- power functions such as $y = \sqrt{x-2}$, $y = 5/x$, $y = 1/x^2$

Great importance is placed on understanding the general and complete appearance of the function graphs studied. Special attention is given to the linear function and the ability to state the equation for a line through two points.

With a graphing calculator

In applications, graphs are plotted with graphing calculators. Good skill in choosing suitable viewing windows for the coordinate system is desirable. Interesting points such as extreme points, zeroes, and points of intersection with axes are found using the calculator's tool-kit.

Integrals

In applications, be able to

- set up an integral
- interpret the meaning of an integral.

Without calculating devices (analytic integration)

Using an antiderivative, be able to calculate by hand very simple integrals for

- polynomials, for example, $f(x) = 5x^5 - x/4$
- trigonometric functions, for example, $f(x) = 3 \sin 4x$
- exponential functions, for example, $f(x) = 4e^{3x}$
- power functions, for example, $f(x) = 5/x$, $f(x) = 3/x^2$, $f(x) = 4\sqrt{x}$,

and to understand that an antiderivative can be verified by differentiation.

With a graphing calculator (numerical integration)

In applied problems, the programs in the calculator are used to calculate definite integrals. The essential part is setting up the integral. Exact calculation of integrals is de-emphasized.

Hans Brolin is professor of mathematics in the Department of Teacher Training at the University of Uppsala, Sweden. He holds a doctorate in pure mathematics, but his primary interest has been research in mathematics education. Lars-Eric Björk holds a master's degree and is currently at Sunnerboskolan in Ljungby, Sweden. He has taught mathematics courses in secondary schools for many years and in 1996 received a doctorate honoris causa *at the University of Uppsala for his work in mathematics education.*